Interpreting Propositional Logic (Part 1)

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- A couple of problems with doing this:
  1. The validity of a complex PL sentence is always dependent on the validity of its component atomic PL sentences. But we can't always know whether all the atomic sentences are true or false!
  2. The syntax of PL is recursive, so a PL sentence can be arbitrarily large. Given any two PL sentences S and T, we can always form $\neg S$, $S \wedge T$, $T \rightarrow S$, etc.

Overview

- To handle problem 1 above, we'll need to consider every possible way things could be.
- That is, given that we can't always know the truth value of each atomic proposition, we need to devise a scheme for discover what the truth value of a complex proposition *would be* just in case we *did* know what the truth values of all its component atomic propositions were.

Enumerating the Possibilities I

- To that end, we look at the simplest case: a single atomic proposition (call it A). Since A is a proposition, it must have a truth value, and so we know there are only two ways things could be (call them w_1 and w_2):

	A
w_1	T
w_2	F

  Here, the **truth assignments** w_1 and w_2 capture all the possible truth values for A: either A is true (w_1) or else it is false (w_2).

Enumerating the Possibilities II

- The next most complicated case is a situation with two atomic propositions A and B. Now we have to consider four separate cases:

	A	B
w_1	T	T
w_2	T	F
w_3	F	T
w_4	F	F

  In this case, both A and B could be true (or false) and A could be true with B false or vice versa.

- This is an instance of a general pattern: each time we consider another atomic proposition, the number of ways things could be doubles. That is, for a sentence of PL containing n atomic propositions, there are 2^n ways things could be.

Example I

- More concretely:

(1)
   a. Pastor Ingqvist likes lutefisk.
   b. Evelyn likes Powdermilk Biscuits.
   c. Florian likes Walleye.

Example II

Let L be the proposition expressed by (1a), P the proposition expressed by (1b), and W the proposition expressed by (1c). Then there are $2^3 = 8$ possible truth assignments:

	L	P	W
w_1	T	T	T
w_2	T	T	F
w_3	T	F	T
w_4	T	F	F
w_5	F	T	T
w_6	F	T	F
w_7	F	F	T
w_8	F	F	F

Example III

- Suppose we happen to know that Evelyn does indeed like Powdermilk Biscuits and Florian really likes Walleye but that Pastor Ingqvist actually can't stand lutefisk. Then the truth assignment w_5 corresponds to how things are in the real world.
- But more generally, we'd like to know what *would have* happened in the other cases.

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- We also know that although there are infinitely many possible complex propositions, there are only finitely many *ways* of connecting atomic propositions to form complex ones (namely, five: $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$).
- So dealing with problem 2 above just means saying what each of the connectives does to the truth values of the proposition(s) (atomic or complex) it is connecting.

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- What fundamental motivating principle of semantics does this scheme remind you of?

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- That is, if a proposition P is true (false), then $\neg P$ is false (true).
- We capture this fact in the **truth table** for negation (shown in Table 1).

φ	$(\neg\varphi)$
T	F
F	T

Table 1: Truth table for negation.

This truth table says that for a given (atomic or complex) PL sentence φ, every truth assignment that assigns T for φ also assigns F for $\neg\varphi$ and vice versa.

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- Negation in PL is used to represent the English usages of negation found in *not*, *it is not the case that*, etc.
- To see if this interpretation of negation corresponds with our intuitions about how language and reasoning interact, consider

  (2) Clint sees Myrtle.

  Let M be the proposition expressed by (2). Then without knowing *whether* M is true or not, we know that *if* M is true then *Clint does not see Myrtle* (i.e., $\neg M$) is false. Likewise, if M is false, then $\neg M$ must be true.

Exercise 1

Given an argument that depends on four distinct atomic propositions, how many possible truth assignments are there for those atomic propositions?

Exercise 2

Assume that a certain argument is based on only four atomic propositions: A, B, C and D. Write out all the possible truth assignments that argument could have.

Exercise 3

Let S be a sentence of PL. To know the truth value of $(\neg S)$, do we have to know what the truth value of S is? Why or why not?