Calculating Truth Conditions

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- Truth tables let us determine the truth value of the propositions connected by a given connective.
- By repeatedly applying truth tables to connectives and the propositions they connect, we can calculate the truth conditions of an arbitrarily complex sentence of PL.

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- Similarly, the first row under $\wedge$ contains an F because one of the conjuncts of $(\neg A) \wedge B$ (namely, $\neg A$) is false under the assignment on the first row, making $(\neg A) \wedge B$ false under that assignment as the truth table for $\wedge$ says.

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- If two or more sentences have the same interpretation on every truth assignment, they are said to be **equivalent**. For example, any two tautologies are equivalent to each other (but *not* equal!).
- If an argument's premises are true in the actual world, we say that the argument is **sound**.

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Table 2: Truth condition calculation for (2).

What is another name for the class of sentences that (2) belongs to?

Exercises I

Problem 1

For each of the following sentences of PL, say what the main connective is:

a. $\neg(A \to B \to C)$
b. $(A \wedge B) \leftrightarrow C$
c. $\neg(\neg A \wedge \neg B)$
d. $(\neg A \wedge \neg B)$
e. $\neg(B \to (A \vee \neg C))$
f. $(\neg B \to (A \vee \neg C))$
g. $\neg A \to (B \wedge (\neg C \leftrightarrow D))$

Exercises II

Problem 2

Construct truth tables that show that de Morgan's laws are indeed tautologies:

- a. $\neg(A \wedge B) \leftrightarrow ((\neg A) \vee (\neg B))$
- b. $\neg(A \vee B) \leftrightarrow ((\neg A) \wedge (\neg B))$

Problem 3

Let φ and ψ be equivalent propositions. What do we know about the interpretation of the sentence $\varphi \leftrightarrow \psi$?

Exercises III

Problem 4

Construct truth tables for the following two sentences:

a. $A \to B$
b. $(\neg B) \to (\neg A)$

Given the truth tables you constructed, how are these sentences related?

Problem 5

Let S be a sound argument. What do we know about the truth value of the conclusion(s) of S?