

# Calculating Truth Conditions

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January 30, 2012

## Using Truth Assignments and Truth Tables

- Truth tables let us determine the truth value of the propositions connected by a given connective.
- By repeatedly applying truth tables to connectives and the propositions they connect, we can calculate the truth conditions of an arbitrarily complex sentence of PL.

### Example Calculation

*Example 1.* We start with a simple case of a binary connective between two atomic sentences of PL.

$$(\neg A) \wedge B \tag{1}$$

We use Table 2 to calculate the truth conditions of (1). The truth values for the main connective  $\wedge$  are in boldface.

$A$	$B$	$(\neg A)$	$\wedge$	$B$
T	T	F	<b>F</b>	
T	F	F	<b>F</b>	
F	T	T	<b>T</b>	
F	F	T	<b>F</b>	

Table 1: Truth condition calculation for (1).

### Example Calculation Explained

- On the left side of the line are the truth assignments for all the atomic propositions contain within (1), namely  $A$  and  $B$ .
- On the right side of the line, we write beneath each connected proposition (namely  $\neg A$  and  $(\neg A) \wedge B$ ) what its truth value would be given the calculated truth values of the propositions it connects.
- For example, the second row beneath  $\neg A$  contains an F because that's what the truth table for negation says the value of  $\neg A$  is under a truth assignment that makes  $A$  true.
- Similarly, the first row under  $\wedge$  contains an F because one of the conjuncts of  $(\neg A) \wedge B$  (namely,  $\neg A$ ) is false under the assignment on the first row, making  $(\neg A) \wedge B$  false under that assignment as the truth table for  $\wedge$  says.

## Some Things to Notice

- Notice that, in Example 1, the entire proposition  $(\neg A) \wedge B$  is only true in the third row, the truth assignment with  $A$  false and  $B$  true.
- Since (1) is sometimes false and sometimes true, depending on the truth assignment chosen, it is called a **contingent** proposition.
- Some sentences (e.g.  $A \vee \neg A$  and  $A \rightarrow A$ ) are true under *every* truth assignment; such sentences are said to express a **tautology** or **logical truth**.
- Sentences that are false under every assignment are called **contradictions** or **logical falsehoods**, for example the negated tautology  $\neg(A \vee \neg A)$ .
- If two or more sentences have the same interpretation on every truth assignment, they are said to be **equivalent**. For example, any two tautologies are equivalent to each other (but *not* equal!).
- If an argument's premises are true in the actual world, we say that the argument is **sound**.

## Another Example Calculation

*Example 2.* A slightly more complex example:

$$(A \wedge (A \rightarrow B)) \rightarrow B \quad (2)$$

We proceed as before, using the truth tables for  $\wedge$  and  $\rightarrow$ : What is another name for the class of sentences

$A$	$B$	$(A \wedge (A \rightarrow B))$	$\rightarrow$	$B$
T	T	T	T	<b>T</b>
T	F	F	F	<b>T</b>
F	T	F	T	<b>T</b>
F	F	F	T	<b>T</b>

Table 2: Truth condition calculation for (2).

that (2) belongs to?

## Homework

### Exercises

**Problem 1.** For each of the following sentences of PL, say what the main connective is:

- $\neg(A \rightarrow B \rightarrow C)$
- $(A \wedge B) \leftrightarrow C$
- $\neg(\neg A \wedge \neg B)$
- $(\neg A \wedge \neg B)$
- $\neg(B \rightarrow (A \vee \neg C))$
- $(\neg B \rightarrow (A \vee \neg C))$
- $\neg A \rightarrow (B \wedge (\neg C \leftrightarrow D))$

**Problem 2.** Construct truth tables that show that de Morgan's laws are indeed tautologies:

a.  $\neg(A \wedge B) \leftrightarrow ((\neg A) \vee (\neg B))$

b.  $\neg(A \vee B) \leftrightarrow ((\neg A) \wedge (\neg B))$

**Problem 3.** Let  $\varphi$  and  $\psi$  be equivalent propositions. What do we know about the interpretation of the sentence  $\varphi \leftrightarrow \psi$ ?

**Problem 4.** Construct truth tables for the following two sentences:

a.  $A \rightarrow B$

b.  $(\neg B) \rightarrow (\neg A)$

Given the truth tables you constructed, how are these sentences related?

**Problem 5.** Let  $S$  be a sound argument. What do we know about the truth value of the conclusion(s) of  $S$ ?