Arguments and Logic

Scott Martin

Jan 17, 2011

Arguments, Premises, and Conclusions

- An **argument** is an application of reasoning to go from already held beliefs to new conclusions.

Arguments, Premises, and Conclusions

- An **argument** is an application of reasoning to go from already held beliefs to new conclusions.
- One way of talking about the structure of arguments is to construe them as sets of propositions expressed by declarative NL sentences with one or more **premises** with one or more **conclusions**.

Arguments, Premises, and Conclusions

- An **argument** is an application of reasoning to go from already held beliefs to new conclusions.
- One way of talking about the structure of arguments is to construe them as sets of propositions expressed by declarative NL sentences with one or more **premises** with one or more **conclusions**.
- Thinking of arguments as a collection of propositions simplifies a lot, but lets us go a long way toward saying what a **valid** argument is.

(1)	Premise	Everyone who lives in Lake Wobegon eats lutefisk.
	Premise	Clarence lives in Lake Wobegon.
	Conclusion	Clarence eats lutefisk.

(1)	Premise	Everyone who lives in Lake Wobegon eats lutefisk.
	Premise	Clarence lives in Lake Wobegon.
	Conclusion	Clarence eats lutefisk.
(2)	Premise	If you worked hard, you are successful.
	Premise	You are successful.
	Conclusion	You worked hard.

(1)	Premise	Everyone who lives in Lake Wobegon eats lutefisk.
	Premise	Clarence lives in Lake Wobegon.
	Conclusion	Clarence eats lutefisk.
(2)	Premise	If you worked hard, you are successful.
	Premise	You are successful.
	Conclusion	You worked hard.
(3)	Premise	Guy Noir takes the elevator to his office.
	Premise	Ralph's Pretty Good Grocery has a sale on dented cans of beans.
	Conclusion	Lefty starts putting milk in his coffee.

- Notice that some of these are valid and some are not, so a collection of propositions (some of which are premises and some conclusions) is not necessarily a valid argument.

- Notice that some of these are valid and some are not, so a collection of propositions (some of which are premises and some conclusions) is not necessarily a valid argument.
- A set of propositions S **entails** a proposition P if and only if, regardless of how things happen to be, if every proposition in S is true then P is true.

- Notice that some of these are valid and some are not, so a collection of propositions (some of which are premises and some conclusions) is not necessarily a valid argument.
- A set of propositions S **entails** a proposition P if and only if, regardless of how things happen to be, if every proposition in S is true then P is true.
- An argument is **deductively valid** if and only if its premises entail its conclusion(s). This is a stronger notion than the notion of truth preservation, which could occur accidentally.

Defining a Logical Language

- One definition of the formal discipline of logic: the study of what makes an argument valid or not.

Defining a Logical Language

- One definition of the formal discipline of logic: the study of what makes an argument valid or not.
- We'll look at a particular variant of logic called **propositional logic (PL)** that's useful for examining arguments based on NL meanings.

Defining a Logical Language

- One definition of the formal discipline of logic: the study of what makes an argument valid or not.
- We'll look at a particular variant of logic called **propositional logic (PL)** that's useful for examining arguments based on NL meanings.
- Like our Card Language, PL has both a syntax (form) and a semantics (interpretation).
- The interaction between PL's syntax and semantics is used to model the interaction between English form and interpretation:

	Form	**Interpretation**
English	(declarative) sentence	proposition
PL	sentence	truth value

Defining a Logical Language

- One definition of the formal discipline of logic: the study of what makes an argument valid or not.
- We'll look at a particular variant of logic called **propositional logic (PL)** that's useful for examining arguments based on NL meanings.
- Like our Card Language, PL has both a syntax (form) and a semantics (interpretation).
- The interaction between PL's syntax and semantics is used to model the interaction between English form and interpretation:

	Form	**Interpretation**
English	(declarative) sentence	proposition
PL	sentence	truth value

- Given that we're already familiar with what English declaratives, propositions, and truth values are, we'll start by saying what counts as a sentence of PL.

PL Syntax Defined

Definition (Syntax of Propositional Logic (PL))

Sentences of PL are subject to the following rules:

1. An uppercase letter (A–Z) is a sentence of PL.

PL Syntax Defined

Definition (Syntax of Propositional Logic (PL))

Sentences of PL are subject to the following rules:

1. An uppercase letter (A–Z) is a sentence of PL.
2. If φ and ψ are PL sentences, then so are
    1. $(\neg\,\varphi)$

PL Syntax Defined

Definition (Syntax of Propositional Logic (PL))

Sentences of PL are subject to the following rules:

1. An uppercase letter (A–Z) is a sentence of PL.
2. If φ and ψ are PL sentences, then so are
    1. $(\neg\,\varphi)$
    2. $(\varphi \wedge \psi)$

PL Syntax Defined

Definition (Syntax of Propositional Logic (PL))

Sentences of PL are subject to the following rules:

1. An uppercase letter (A–Z) is a sentence of PL.
2. If φ and ψ are PL sentences, then so are
   1. $(\neg \varphi)$
   2. $(\varphi \wedge \psi)$
   3. $(\varphi \vee \psi)$

PL Syntax Defined

Definition (Syntax of Propositional Logic (PL))

Sentences of PL are subject to the following rules:

1. An uppercase letter (A–Z) is a sentence of PL.
2. If φ and ψ are PL sentences, then so are
    1. $(\neg\, \varphi)$
    2. $(\varphi \wedge \psi)$
    3. $(\varphi \vee \psi)$
    4. $(\varphi \rightarrow \psi)$

PL Syntax Defined

Definition (Syntax of Propositional Logic (PL))

Sentences of PL are subject to the following rules:

1. An uppercase letter (A–Z) is a sentence of PL.
2. If φ and ψ are PL sentences, then so are
   1. $(\neg\varphi)$
   2. $(\varphi \wedge \psi)$
   3. $(\varphi \vee \psi)$
   4. $(\varphi \rightarrow \psi)$
   5. $(\varphi \leftrightarrow \psi)$

PL Syntax Defined

Definition (Syntax of Propositional Logic (PL))

Sentences of PL are subject to the following rules:

1. An uppercase letter (A–Z) is a sentence of PL.
2. If φ and ψ are PL sentences, then so are
    1. $(\neg \varphi)$
    2. $(\varphi \wedge \psi)$
    3. $(\varphi \vee \psi)$
    4. $(\varphi \rightarrow \psi)$
    5. $(\varphi \leftrightarrow \psi)$
3. Nothing else is a sentence of PL.

Some Elaboration

- The letters A–Z are sometimes called **sentence letters**.

Some Elaboration

- The letters A–Z are sometimes called **sentence letters**.
- The symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ are sometimes called **(truth-functional) connectives**.

Some Elaboration

- The letters A–Z are sometimes called **sentence letters**.
- The symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ are sometimes called **(truth-functional) connectives**.
- The **scope** of a connective is the smallest sentence that contains that connective; the connective with the widest scope in a given sentence is the **main connective** of that sentence.

Some Elaboration

- The letters A–Z are sometimes called **sentence letters**.
- The symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ are sometimes called **(truth-functional) connectives**.
- The **scope** of a connective is the smallest sentence that contains that connective; the connective with the widest scope in a given sentence is the **main connective** of that sentence.
- Everything we'll encounter in PL will be entirely made up of sentence letters, connectives, and parentheses.

Some Elaboration

- The letters A–Z are sometimes called **sentence letters**.
- The symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ are sometimes called **(truth-functional) connectives**.
- The **scope** of a connective is the smallest sentence that contains that connective; the connective with the widest scope in a given sentence is the **main connective** of that sentence.
- Everything we'll encounter in PL will be entirely made up of sentence letters, connectives, and parentheses.
- To keep clutter to a minimum, we'll sometimes omit parentheses when they are not needed because no ambiguity can arise. So for example, we'll sometimes write $\neg A$ instead of $(\neg A)$, $A \rightarrow B$ instead of $(A \rightarrow B)$, etc.

Propositions

- There are two flavors of propositions: **atomic** (most basic) and **complex** (built up out of multiple atomic propositions).

Propositions

- There are two flavors of propositions: **atomic** (most basic) and **complex** (built up out of multiple atomic propositions).
- An atomic proposition is the smallest unit of meaning that has a truth value.

Propositions

- There are two flavors of propositions: **atomic** (most basic) and **complex** (built up out of multiple atomic propositions).
- An atomic proposition is the smallest unit of meaning that has a truth value.
- This distinction parallels NL:

(4) a. LuAnne goes to the Loons game.

b. Florian goes to Buck's Rent-a-Tux.

c. Either Florian goes to Buck's Rent-a-Tux or LuAnne goes to the Loons game.

(5) a. The Sidetrack Tap is open.

b. Clint stops for a beer.

c. If the Sidetrack Tap is open then Clint stops for a beer.

Propositions, cont.

- The complex proposition expressed by (4c) is simply a combination of the two atomic propositions expressed by (4a) and (4b). Similarly (5c) expresses a conditional proposition made up of (5a) and (5b).

Propositions, cont.

- The complex proposition expressed by (4c) is simply a combination of the two atomic propositions expressed by (4a) and (4b). Similarly (5c) expresses a conditional proposition made up of (5a) and (5b).
- Correspondingly, all of (4a), (4b), (5a), and (5b) are representable in PL as atomic propositions, but (4c) and (5c) cannot be–they must be represented as complex propositions built using connectives.

Propositions, cont.

- The complex proposition expressed by (4c) is simply a combination of the two atomic propositions expressed by (4a) and (4b). Similarly (5c) expresses a conditional proposition made up of (5a) and (5b).
- Correspondingly, all of (4a), (4b), (5a), and (5b) are representable in PL as atomic propositions, but (4c) and (5c) cannot be–they must be represented as complex propositions built using connectives.
- This distinction between atomic and complex propositions, along with the recursive nature of the rules for building complex propositions, is meant to capture the fact that NL syntax is both recursive and compositional. Since sentences can contain other sentences, some PL propositions (namely, complex ones) contain other propositions.

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.
  - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.

Differences between PL and NL

- But PL differs from NL in that:
    - Different propositions are represented using different propositional letters.
    - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.
    - Each syntactically well formed sentence of PL has a single well defined interpretation (that we'll discuss later).

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.
  - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.
  - Each syntactically well formed sentence of PL has a single well defined interpretation (that we'll discuss later).
- Our inventory of connectives is meant to mirror (certain uses of) so-called "logic words" in English:

English	**PL**
not	$\neg$

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.
  - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.
  - Each syntactically well formed sentence of PL has a single well defined interpretation (that we'll discuss later).
- Our inventory of connectives is meant to mirror (certain uses of) so-called "logic words" in English:

English	**PL**
not	$\neg$
and	$\wedge$

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.
  - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.
  - Each syntactically well formed sentence of PL has a single well defined interpretation (that we'll discuss later).
- Our inventory of connectives is meant to mirror (certain uses of) so-called "logic words" in English:

English	**PL**
not	$\neg$
and	$\wedge$
or	$\vee$

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.
  - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.
  - Each syntactically well formed sentence of PL has a single well defined interpretation (that we'll discuss later).
- Our inventory of connectives is meant to mirror (certain uses of) so-called "logic words" in English:

English	**PL**
not	$\neg$
and	$\wedge$
or	$\vee$
if ... then ...	$\rightarrow$

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.
  - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.
  - Each syntactically well formed sentence of PL has a single well defined interpretation (that we'll discuss later).
- Our inventory of connectives is meant to mirror (certain uses of) so-called "logic words" in English:

English	**PL**
not	$\neg$
and	$\wedge$
or	$\vee$
if ... then ...	$\rightarrow$
if and only if	$\leftrightarrow$

Differences between PL and NL

- But PL differs from NL in that:
  - Different propositions are represented using different propositional letters.
  - The syntactic form of PL sentences do not give rise to ambiguity the way that we seen in NL sentences.
  - Each syntactically well formed sentence of PL has a single well defined interpretation (that we'll discuss later).
- Our inventory of connectives is meant to mirror (certain uses of) so-called "logic words" in English:

English	**PL**
not	$\neg$
and	$\wedge$
or	$\vee$
if ... then ...	$\rightarrow$
if and only if	$\leftrightarrow$

In fact, the PL connectives are often pronounced using their English counterparts when reading PL formulas out loud.

Exercise 1

Each of the following sentences expresses a proposition. For each, say whether that proposition is atomic or complex, and if it's complex, what atomic propositions it's built from:

1. Clarence goes to the Mercantile and then eats lunch.
2. The Norwegian bachelor farmers all went to church.
3. If Myrtle is at the Chatterbox Cafe, it must be Wednesday.
4. Clint, Wally, or Pastor Ingqvist were at the 5 and dime.
5. Evelyn gets ready for the after-work rush.
6. Carl Krebsback doesn't read the Herald Star.

Exercise 2

Using the definition on page 15, say which of the following are sentences of PL and which are not. (Parentheses are sometimes omitted for clarity.)

1. $\neg\neg Z$
2. $A \rightarrow$
3. $A \wedge B$
4. $\vee D$
5. $E \vee (C \wedge \neg A)$
6. $((C \rightarrow D) \wedge C) \rightarrow D$
7. $((B \leftrightarrow \neg C \vee A \leftrightarrow C) \rightarrow A) \leftrightarrow B$
8. $(M \wedge (C \wedge \leftrightarrow N)) \rightarrow E \wedge F$
9. $Q \neg$
10. $\neg(Q \rightarrow P)$

Exercise 3

Write 5 sentences of propositional logic that each contain at least 3 symbols that are not parentheses.