Linguistics 280: Problem Set 4

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Instructions. *Complete these problems by noon on Wednesday, March 14, 2012. Since we won't have class, you have the option of either emailing me an electronic version, uploading an electronic version to the Carmen dropbox, or printing it and delivering it to my mailbox in 225 Oxley Hall. As always, all submitted work must be your own.*

Problem 1. Explain how the rule of Implication Introduction reflects the entailment present in a valid argument.

Problem 2. How does the notation used in a natural deduction (ND) sequent capture the notion of a deductively valid argument as we formally defined it?

Problem 3. Given the choice of using non-sequent-style ND (the first way we did things) and sequent-style ND (the second way we did things) to describe a deductively valid argument, which one would you choose? Why? (There's not really a "right" answer here, I'm just looking for good reasons that support your choice.)

Problem 4. What is special about a proof that doesn't rely on any premises? That is, what does it mean when you have a proof that just has a conclusion?

$$\dfrac{\dfrac{\dfrac{(A\wedge B)\to C\vdash (A\wedge B)\to C \qquad \dfrac{A\vdash A \qquad B\vdash B}{A,B\vdash A\wedge B}\,(\wedge\mathrm{I})}{(A\wedge B)\to C, A, B\vdash C}\,(\to\mathrm{E})}{(A\wedge B)\to C, A\vdash B\to C}\,(\to\mathrm{I})}{(A\wedge B)\to C\vdash A\to(B\to C)}\,(\to\mathrm{I})$$

Figure 1: Proof used in Problem 5.

Problem 5. Consider the proof in Figure 1. Then, do the following:

a. Re-write this proof using the non-sequent-style ND we first looked at.

b. Say in words what this proof is demonstrating graphically.

c. Explain what steps would be necessary to show that $(A\wedge B)\to C$ is *equivalent* to $A\to(B\to C)$. (This equivalence is usually referred to as **(un-)Currying**.)

Problem 6. Consider the argument in (1):

(1) a. Wally eats rhubarb pie. Evelyn and Myrtle both eat rhubarb pie.
b. Wally and Evelyn both eat rhubarb pie. Myrtle eats rhubarb pie.
c. Wally, Evelyn, and Myrtle all eat rhubarb pie.

Notice that no matter which of (1a) or (1b) we start with, the conclusion (1c) holds. That is, we can deduce (1c) based on (1a) alone or (1b) alone. Assume the following translations for the propositions in (1):

W Wally eats rhubarb pie.

E Evelyn eats rhubarb pie.

M Myrtle eats rhubarb pie.

then show that (1a) and (1b) are equivalent, that is, that the PL sentences $W \wedge (E \wedge M)$ and $(W \wedge E) \wedge M$ are equivalent to each other (this equivalence is called the **associativity of** $\wedge$).

Problem 7 (Bonus)**.** Do the second half of the proof in Figure 1. That is, show that $(A \wedge B) \to C$ is equivalent to $A \to (B \to C)$.

Problem 8 (Bonus)**.** Give a proof that assumes both $A \to B$ and $B \to C$, then uses those assumptions to prove that $A \to C$ holds (this entailment is called the **transitivity of** $\to$).

Problem 9 (Bonus)**.** Show that $(A \to B) \wedge (A \to C)$ is equivalent to $A \to (B \wedge C)$.