Calculating Truth Conditions

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Using Truth Assignments and Truth Tables

- Truth tables let us determine the truth value of the propositions connected by a given connective.
- By repeatedly applying truth tables to connectives and the propositions they connect, we can calculate the truth conditions of an arbitrarily complex sentence of PL.

Example Calculation

Example 1. We start with a simple case of a binary connective between two atomic sentences of PL.

$$(\neg A) \wedge B \tag{1}$$

We use Table 1 to calculate the truth conditions of (1). The truth values for the main connective $\wedge$ are in boldface.

A	B	$(\neg A)$	$\wedge$	B
T	T	F	**F**	T
T	F	F	**F**	F
F	T	T	**T**	T
F	F	T	**F**	F

Table 1: Truth condition calculation for (1).

Example Calculation Explained

- On the left side of the line are the truth assignments for all the atomic propositions contain within (1), namely A and B.
- On the right side of the line, we write beneath each connected proposition (namely $\neg A$ and $(\neg A) \wedge B$) what its truth value would be given the calculated truth values of the propositions it connects.
- For example, the second row beneath $\neg A$ contains an F because that's what the truth table for negation says the value of $\neg A$ is under a truth assignment that makes A true.
- Similarly, the first row under $\wedge$ contains an F because one of the conjuncts of $(\neg A) \wedge B$ (namely, $\neg A$) is false under the assignment on the first row, making $(\neg A) \wedge B$ false under that assignment as the truth table for $\wedge$ says.

Some Things to Notice

- Notice that, in Example 1, the entire proposition $(\neg A) \wedge B$ is only true in the third row, the truth assignment with A false and B true.
- Since (1) is sometimes false and sometimes true, depending on the truth assignment chosen, it is called a **contingent** proposition.
- Some sentences (e.g. $A \vee \neg A$ and $A \rightarrow A$) are true under *every* truth assignment; such sentences are said to express a **tautology** or **logical truth**.
- Sentences that are false under every assignment are called **contradictions** or **logical falsehoods**, for example the negated tautology $\neg(A \vee \neg A)$.
- If two or more sentences have the same interpretation on every truth assignment, they are said to be **equivalent**. For example, any two tautologies are equivalent to each other (but *not* equal!).
- If an argument's premises are true in the actual world, we say that the argument is **sound**.

Homework

Exercises

Problem 1. For each of the following sentences of PL, say what the main connective is:

a. $\neg(A \rightarrow B \rightarrow C)$

b. $(A \wedge B) \leftrightarrow C$

c. $\neg(\neg A \wedge \neg B)$

d. $(\neg A \wedge \neg B)$

e. $\neg(B \rightarrow (A \vee \neg C))$

f. $(\neg B \rightarrow (A \vee \neg C))$

g. $\neg A \rightarrow (B \wedge (\neg C \leftrightarrow D))$

Problem 2. Construct truth tables that show that de Morgan's laws are indeed tautologies:

a. $\neg(A \wedge B) \leftrightarrow ((\neg A) \vee (\neg B))$

b. $\neg(A \vee B) \leftrightarrow ((\neg A) \wedge (\neg B))$

Problem 3. Let φ and ψ be equivalent propositions. What do we know about the interpretation of the sentence $\varphi \leftrightarrow \psi$?

Problem 4. Construct truth tables for the following two sentences:

a. $A \rightarrow B$

b. $(\neg B) \rightarrow (\neg A)$

Given the truth tables you constructed, how are these sentences related?

Problem 5. Let S be a sound argument. What do we know about the truth value of the conclusion(s) of S?