Problem Set Eight

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Linguistics 680

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These problems are due Tue. Nov. 29.

Problem 1

Give a natural-deduction proof tree for the following theorem of pure LL, known as **Composition** or **Generalized Contraposition**:

$$\vdash (A \multimap B) \multimap (B \multimap C) \multimap (A \multimap C)$$

Problem 2

Give a natural-deduction proof tree for the following theorem of pure LL, known as **Geach's Law**:

$$\vdash (A \multimap B) \multimap (C \multimap A) \multimap (C \multimap B)$$

Problem 3

Give a natural-deduction proof tree for the following theorem of pure PIPL:

$$A \vdash B \rightarrow A$$

Problem 4

Give a natural-deduction proof tree for the following theorem of pure PIPL:

$$A \rightarrow A \rightarrow B \vdash A \rightarrow B$$

Problem 5

Show that a subset S of a preorder P is upper closed iff $P \setminus S$ is lower closed.

Problem 6

Show that any nonempty intersection of filters is a filter.

Problem 7

Show that for any element p of a lower presemilattice, $\uparrow p$ is a filter.

Problem 8

Show that a filter is principal iff it has a least element.

Problem 9

Show that a filter F in a prelattice P is prime iff $F \neq P$ and for all $p, q \in P$, if $p \sqcup q \in F$ then either $p \in F$ or $q \in F$.

Problem 10

Let D be a set and A its powerset. Since A is a boolean algebra, we can ask whether a given subset of A is upper- or lower- closed, meet- or join-closed, a filter or an ideal, etc. Now let S be a subset of D. (Intuitively, think of D as the set of *individuals*, and think of S as the extension of some property of individuals in the actual world, e.g. the set of donkeys, the set of braying things, etc.) Now (following, more or less, the practice of extensional Montague semantics) define the following subsets of A:

a. $\mathsf{every}(S) =_{\mathrm{def}} \{T \subseteq D \mid S \subseteq T\}$

b. $\mathsf{some}(S) =_{\mathrm{def}} \{T \subseteq D \mid S \cap T \neq \emptyset\}$

c. $\mathsf{no}(S) =_{\mathrm{def}} \{T \subseteq D \mid S \cap T = \emptyset\}$

d. $\mathsf{only}(S) =_{\mathrm{def}} \mathsf{some}(S) \cap \mathsf{no}(D \setminus S)$

a. Show that $\mathsf{every}(S)$ is a principal filter, and give a necessary and sufficient condition for it to be an ultrafilter.

b. Show that $\mathsf{some}(S)$ is upper-closed. Under what conditions is it a filter?

c. Show that $\mathsf{no}(S)$ is a principal ideal. What is its generator?

d. Is $\mathsf{only}(S)$ lower-closed?