Problem Set Five: Inductive Proof and Infinity

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These problems are due Tue. Nov. 1. Most of these problems were mentioned as exercises in *FFLT* or in the slides/handouts, so be sure not to overlook any hints given there.

Problem 1

Prove the commutativity of addition.

Problem 2

Prove that for any binary relation R on A, R^+ is transitive.

Problem 3

Prove that every natural number is a transitive set.

Problem 4

Prove that no natural number is Dedekind infinite.

Problem 5

a. Prove the corollary that no finite set is Dedekind infinite.

b. Prove the corollary that any Dedekind infinite set is infinite.

Problem 6

Prove the corollary that no two distinct natural numbers are equinumerous.

Problem 7

Prove the corollary that for any finite set A, there is a unique natural number equinumerous with A.

Problem 8

Prove that for any sets A, B, and C:

a. $A \preceq A$;

b. if $A \preceq B$ and $B \preceq C$ then $A \preceq C$; and

c. $A \preceq \wp(A)$.

Problem 9

a . Prove the corollary that any countably infinite set is equinumerous with ω.

b. Prove the corollary that any infinite subset of ω is equinumerous with ω.

Problem 10

Prove that $\wp(\omega)$ is nondenumerable.