Problem Set Two: Mathese and the Natural Numbers

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Background

In chapter 4, we will consider some consequences of modelling the *natural numbers* within our set theory. But we already have enough background to begin developing the basic concepts involved in that enterprise. In these problems, we do just that, by way of getting some practice with Mathese.

Problem 1

We will define a set x to be **inductive** iff x has 0 as a member and has the successor of each of its members as a member. Express this condition on x in Mathese (or, if you prefer, in FOL).

Problem 2

We will define a set x to be a **natural number** iff it belongs to every inductive set. Express this condition on x in Mathese (or, if you prefer, in FOL). [Hint: now that the predicate **inductive** has been defined, you can abbreviate 'x is inductive' by '**inductive**(x)'.]

Problem 3

We will assume (Assumption 7) that there is a set whose members are the natural numbers. State this assumption in Mathese (or, if you prefer, in FOL.) [Hint: now that the predicate **natural number** has been defined, you can abbreviate 'x is a natural number' by '**nat**(x)'.]

Problem 4

Now that we have assumed there is a set whose members are the natural numbers, we can give that set a name: ω. What is the justification fior doing this?

Problem 5

Prove Theorem 4.1, that w is inductive. [Note: this, and the proofs that follow, can be in plain English, Mathese, or a combination of the two. Just make sure the proofs are clear and unambiguous, and expressed in complete sentences.]

Problem 6

Prove Theorem 4.2. that w is a subset of every inductive set.

Problem 7

Prove Theorem 4.3 (the Principle of Mathematical Induction), that the only inductive subset of ω is ω.