Calculating Truth Conditions

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August 5, 2010

Using Truth Assignments and Truth Tables

- Truth tables let us determine the truth value of the propositions connected by a given connective.
- By repeatedly applying truth tables to connectives and the propositions they connect, we can calculate the truth conditions of an arbitrarily complex sentence of PL.

Example 1. We start with a simple case of a binary connective between two atomic sentences of PL.

$$(\neg A) \land B \tag{1}$$

We use Table 1 to calculate the truth conditions of (1). On the left side of the line are the truth

A	B	$(\neg A)$	$\land$	B
T	T	F	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	F

Table 1: Truth condition calculation for (1).

assignments for all the atomic propositions contain within (1), namely A and B. On the right side of the line, we write beneath each connected proposition (namely $\neg A$ and $(\neg A) \land B$) what its truth value would be given the calculated truth values of the propositions it connects.

For example, the second row beneath $\neg A$ contains an F because that's what the truth table for negation says the value of $\neg A$ is under a truth assignment that makes A true. Similarly, the first row under $\land$ contains an F because one of the conjuncts of $(\neg A) \land B$ (namely, $\neg A$) is false under the assignment on the first row, making $(\neg A) \land B$ false under that assignment as the truth table for $\land$ says.

- Notice that, in Example 1, the entire proposition $(\neg A) \land B$ is only true in the third row, the truth assignment with A false and B true.
- Since (1) is sometimes false and sometimes true, depending on the truth assignment chosen, it is called a **contingent** proposition.

- Some sentences (e.g. $A \vee \neg A$ and $A \rightarrow A$) are true under *every* truth assignment; such sentences are said to express a **tautology** or **logical truth**.
- Sentences that are false under every assignment are called **contradictions** or **logical falsehoods**, for example the negated tautology $\neg(A \vee \neg A)$.
- If two or more sentences have the same interpretation on every truth assignment, they are said to be **equivalent**. For example, any two tautologies are equivalent to each other (but *not* equal!).
- If an argument's premises are true in the actual world, we say that the argument is **sound**.

Homework

Problem 1. For each of the following sentences of PL, say what the main connective is:

a. $\neg(A \rightarrow B \rightarrow C)$

b. $(A \wedge B) \leftrightarrow C$

c. $\neg(\neg A \wedge \neg B)$

d. $(\neg A \wedge \neg B)$

e. $\neg(B \rightarrow (A \vee \neg C))$

f. $(\neg B \rightarrow (A \vee \neg C))$

g. $\neg A \rightarrow (B \wedge (\neg C \leftrightarrow D))$

Problem 2. Construct truth tables that show that de Morgan's laws are indeed tautologies:

a. $\neg(A \wedge B) \leftrightarrow ((\neg A) \vee (\neg B))$

b. $\neg(A \vee B) \leftrightarrow ((\neg A) \wedge (\neg B))$

Problem 3. Let φ and ψ be equivalent propositions. What do we know about the interpretation of the sentence $\varphi \leftrightarrow \psi$?

Problem 4. Construct truth tables for the following two sentences:

a. $A \rightarrow B$

b. $(\neg B) \rightarrow (\neg A)$

Given the truth tables you constructed, how are these sentences related?

Problem 5. Let S be a sound argument. What do we know about the truth value of the conclusion(s) of S?