Hyperintensional Dynamic Semantics*

Analyzing Definiteness with Enriched Contexts

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Abstract. We present a dynamic semantic theory formalized in higher order logic that synthesizes aspects of de Groote's continuation-based dynamics and Pollard's hyperintensional semantics. In this theory, we rely on an enriched notion of discourse context inspired by the work of Heim and Roberts. We show how to use this enriched context to improve on de Groote's treatment of English definite anaphora by modeling it as presupposition fulfillment.

Keywords: discourse, context, presupposition, definite anaphora, higher order logic

1 Introduction

As Muskens [14, 15] showed, many of the insights of dynamic semantic theories such as Kamp's discourse representation theory (DRT, [9, 10]) and Heim's file change semantics (FCS, [6, 7]) can be formalized within the well-understood framework of classical higher order logic (HOL) of Church [1], Henkin [8], and Gallin [2] that is familiar to Montague semanticists. Also working within HOL, de Groote [4] showed that the description of context update could be streamlined by modeling right contexts by analogy with the *continuations* employed in programming language semantics [27].

Though both Muskens' and de Groote's work are positive developments in the sense of helping to integrate dynamic notions into mainstream semantic theory, both fall short in modeling how definite anaphora works in discourse, which was one of the two central problems that Kamp and Heim originally set out to solve. (The other was to characterize the novelty of indefinite descriptions.) For Muskens, definite pronouns are simply ambiguous with respect to which accessible and sortally appropriate discourse referent they 'pick up'. The trouble with this theory is that, empirically, definite anaphora is generally *not* ambiguous; if it were, it would fail to serve its communicative purpose.

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De Groote fares no better. On his account, the antecedent of a pronoun is picked out by an oracular choice function `sel` from among the sortally correct candidate entities (de Groote does not have discourse referents as distinct from entities in his theory). At first one might think the weakness of this account is that it doesn't tell us anything about *what* choice function this oracle actually is. But in fact it is worse than that, because it is easy to show that *no* choice function is the right one. That is because, in general, the antecedent of a definite pronoun fails to be uniquely determined by the set of sortally appropriate candidate entities. Consider, e.g., the following two narratives:

(A)
1. A donkey and a mule walked in.
2. The donkey was sad.
3. It brayed.

(B)
1. A donkey and a mule walked in.
2. The mule was sad.
3. It brayed.

In both (A) and (B), the set of candidate antecedents consists of two nonhuman entities, a donkey and a mule. In each narrative, one of the candidates has been rendered more salient by virtue of having been re-invoked by a definite description. And in each narrative, the anaphora resolves to the more salient, 'definitized' entity. But the choice function only 'knows about' the members of the set of candidates and their sortal properties, not about their relative salience in the discourse at hand. So it cannot pick the right antecedent both times.

In our view, the weaknesses of Muskens' and de Groote's theories arise because they fail to build in a notion of context that is sufficiently rich to support a satisfactory account of *presuppositions*, the conditions on contexts that must be satisfied in order for utterances to be felicitous. In this paper, we suggest a revision and extension of de Groote's theory that copes with definite anaphora by building in a (slightly) more articulated discourse model inspired by proposals due to Craige Roberts [20]. However, the work reported in this paper is part of a larger research program, joint with Roberts and E. Allyn Smith, aimed at constructing formally explicit, categorially-based, natural language grammars that deal effectively with projective aspects of meaning [22]. Besides Roberts' work on modeling discourse contexts, this research program also builds on Pollard's [19, 18] hyperintensional semantics (relevant aspects of which are sketched below), and on 'pheno-tecto grammar' (PTG; [17, 3, 16, 24]) the line of development in categorial grammar that distinguishes concrete syntax from combinatorics (not touched upon in this paper).

The rest of this paper is organized as follows. We present some facts about English definiteness presuppositions in Sect. 2. In Sect. 3, we formally lay out our hyperintensional dynamic semantic theory of discourse. Specific machinery for dealing with definiteness is introduced in Sect. 4 that handles some of the cases introduced in Sect. 2. Section 5 concludes and promises future work.

2 Facts about Definiteness Presuppositions in English

We take discourse contexts to be comprised of information that is mutual to the interlocutors participating in a discourse, made available either linguistically (i.e. what has been previously uttered by the interlocutors) or non-linguistically (i.e. sense data and world knowledge). Our view of presuppositions is influenced by the work of Stalnaker [25] and later Heim [6, 7] in which the presuppositions of a sentence are taken to be those conditions that must be met by any discourse context in which it is to be felicitously interpreted. For Stalnaker, these conditions are based on what the interlocutors are able to infer from the context. Similarly, Heim formally expresses this notion by modeling discourse contexts as the conjunction of the propositions asserted by the interlocutors, and dynamic meanings as partial functions from contexts to contexts. The domains of these partial functions are determined by the presuppositions of the sentences they interpret.

A wide range of presuppositional phenomena has been discussed in the philosophical and linguistic literature, including (to name just a few) factivity (i.e. verbs like *suck*, *know* and *realize* that take sentences as their complements), presupposition 'projection' and 'cancellation', and the presuppositions associated with definite descriptions (i.e. expressions such as proper names, the English definite article *the*, and pronouns such as *it* that presuppose that a suitable anaphoric antecedent is available in the discourse context in which they occur [21]). Here, we consider only presuppositions of **definiteness**, which encompasses—in senses to be made precise—familiarity and unique greatest salience. To take the most simple example, if

(C) # It brayed

is uttered out of the blue and in the absence of some perceptible nonhuman entity in the immediate surroundings that might plausibly have brayed, the presupposition of the pronoun is not fulfilled and so the utterance is infelicitous. Such a familiarity presupposition need not be globally satisfied, but instead can be satisfied locally (roughly: within the scope of one or more operators) as in

(D) No donkey denies it brays.

For this reason the familiarity presuppositions associated with definite anaphora are usually taken to have to do with the availability not of entities *per se* but rather of "discourse referents", a notion which the ambient theory must make precise.

Definites also presuppose more than just the familiarity of the intended discourse referent:

(E) 1. I saw the donkey.
    2. What donkey?
    3. # Oh, just some donkey out in a field on the way to Upper Sandusky.

Even if the utterances in (E) are situated within a discourse where donkeys have been mentioned or in the presence of one or more donkeys that could serve as resolution targets for *the donkey*, the use of the definite article is infelicitous unless there is one that is uniquely most salient among all the others in the discourse context.

Another dimension to uniqueness is related to the fact that both *it* and *the* also presuppose as their respective anaphoric antecedents a discourse referent that meets certain sortal restrictions. The discourses in (F) and (G) demonstrate these presuppositions:

(F)
1. A donkey had a red blanket.
2. A mule had a blue blanket.
3. $\left\{\begin{array}{l}\ \ \text{The donkey}\\ \#\ \text{It}\end{array}\right\}$ snorted.

(G)
1. A donkey had a red blanket.
2. Another donkey had a blue blanket.
3. $\left\{\begin{array}{l}\ \ \text{The donkey with the blue blanket}\\ \#\ \text{The donkey}\\ \#\ \text{It}\end{array}\right\}$ snorted.

In (F), salience alone is not enough to decide whether *a donkey* or *a mule* antecedes the pronoun *it*. The identity of the antecedent must also be uniquely determinable from the discourse context, but since *it* can be anteceded by any nonhuman entity in English, the noun phrase *the donkey* is used instead to unambiguously single out the pronoun's unique antecedent. The discourse in (G) is a variant of (F) where the property of being a donkey is not enough to disambiguate the antecedent because of the uniqueness presuppositions associated with *the* and *it*. These examples show that sortal restrictions, even potentially complex ones, play a role in determining the unique greatest salience of DRs when they are considered as targets for anaphora resolution.

Finally, the absurd discourse in (H) shows that the definite article identifies not just a familiar and uniquely most salient individual, but one with a certain specified property:

(H)
1. I saw the donkey.
2. What donkey?
3. # That llama we always see on the way to Findlay.

Here, it is infelicitous to use the noun phrase *the donkey* to identify the llama even if it is the most salient individual in the utterance context because it does not have the property of being a donkey.

3 Hyperintensional Dynamic Semantics

Our formalization of discourse dynamics builds on the hyperintensional theory of (static) meaning given by Pollard [19, 18]. Like Montague semantics [13], this semantic theory is couched in HOL and has a basic type e for entities as well as the truth-value type t provided by the underlying logic.[1]

But unlike Montague semantics, we follow Thomason [28] in assuming a basic type p for (static) propositions (but no basic type for worlds).[2] Following Lambek and Scott [11], we also assume (1) a natural number type ω; (2) the type constructors U (unit type) and $\times$ (cartesian product) in addition to the usual $\rightarrow$ (exponential); and (3) separation-style subtyping.[3] Subtypes are usually written in the form $\{x \in T \mid \varphi[x]\}$ where $\varphi[x]$ is a formula (boolean term) possibly with x free. Additionally, we make use of dependent coproduct types parameterized by the natural number type, written $\coprod_n T_n$.

From the typed lambda calculus that underlies the HOL, we have the usual pairing and projection functions; applications are written $(f\, a)$ rather than $f(a)$. Successive applications associate to the left; e.g $(f\ a\ b)$ abbreviates $((f\ a)\ b)$.

The type of propositions is axiomatized as a preboolean algebra (like a boolean algebra, but without antisymmetry), preordered by the entailment relation $\mathsf{entails} : \mathrm{p} \rightarrow \mathrm{p} \rightarrow \mathrm{t}$. The propositional connectives and quantifiers are written as boldface versions of the usual (boolean) connectives of the underlying logic: i.e. $\boldsymbol{\neg}$, $\boldsymbol{\wedge}$, $\boldsymbol{\vee}$, $\boldsymbol{\rightarrow}$, $\boldsymbol{\exists}$, and $\boldsymbol{\forall}$; true denotes a greatest element relative to the entailment preorder (a necessary truth).

Following Heim, we use natural numbers (type ω) as discourse referents (hereafter, DRs). The type ω is equipped with the usual linear order $<$ and the successor function $\mathbf{suc} : \omega \rightarrow \omega$. Additionally, for each natural number n, we define the type of the first n natural numbers as a subtype of ω:

$$\omega_n =_{\mathrm{def}} \{i \in \omega \mid i < n\}$$

These types will be used for the domains of anchors (functions from DRs to entities; we eschew the term 'assignment function' commonly used in the linguistic literature because arguments of these functions are not object-language variables).

We adopt the convention that applications and pairings associate to the left and abstractions associate to the right. Parentheses are sometimes abbreviated using . in the usual way (e.g., $\lambda_x.M$) or omitted altogether when no confusion can arise. When a term contains multiple embedded λ-abstractions of the form $\lambda_a \lambda_b \lambda_c M$, we collapse them together as $\lambda_{abc} M$.

[1] For expository simplicity, we depart from Pollard in not distinguishing between the extensional type e and the corresponding hyperintensional type i (individual concepts).

[2] Using separation subtyping, we can define the type of worlds as a certain subtype of the type $\mathrm{p} \rightarrow \mathrm{t}$ (sets of propositions), but this will not be needed here.

[3] Thus if A is a type and a an A-predicate (closed term of type $A \rightarrow \mathrm{t}$), then there is a type A_a interpreted as the subset of the interpretation of A that has the interpretation of a as its characteristic function; and there is a constant μ_a that denotes the subset embedding.

3.1 Information Structures

To advance from static to dynamic semantics, we need to extend our ontology to model contexts. Our notion of context is a simplified version of Roberts' *discourse information structures* [20], which are in turn inspired by the work of Lewis [12], here called simply *structures*.[4] A structure is a tuple consisting of (1) an *anchor* of entities to a set of DR's, (2) a salience preorder on those DR's called the *resolution* preorder (so-called because it will be used to resolve definite anaphora), and (3) a proposition, the *common ground*, which (following Stalnaker [26]) is the conjunction of all the propositions that are taken by the interlocutors to be mutually agreed upon. The common ground includes not only propositions explicitly asserted and accepted in the discourse, but also encyclopedic knowledge about the world that is assumed as shared background.

To make this notion of structure more precise, we begin by defining the type of n-ary anchors α_n to be the type of functions from the first n discourse referents to entities:

$$\alpha_n =_{\text{def}} \omega_n \rightarrow \text{e}$$
$$\alpha =_{\text{def}} \coprod_n \alpha_n$$

An n-ary anchor can be extended to include a new DR mapped to a specified entity using the function $\bullet_n : \alpha_n \rightarrow \text{e} \rightarrow \alpha_{(\textbf{suc}\ n)}$ (written infix), that is subject to the axiom schema

$$\vdash \forall_{n:\omega} \forall_{a:\alpha_n} \forall_{x:\text{e}} \forall_{m:\omega_{(\textbf{suc}\ n)}} .(a \bullet_n x)\ m = \begin{cases} x & \text{if } m = n \\ (a\ m) & \text{otherwise} \end{cases}$$

To track the relative salience of the DRs in the domain ω_n of an anchor, we use a preorder on ω_n. For arbitrary $n : \omega$, an n-ary *resolution* is just a preorder (reflexive, transitive relation) on the set of DRs:

$$\rho_n =_{\text{def}} \{r \in \omega_n \rightarrow \omega_n \rightarrow \text{t} \mid (\mathsf{preorder}_n\ r)\}$$

Note that this is a subtype of the type of binary relations on ω_n; here $(\mathsf{preorder}_n r)$ is a formula which says of the binary relation r on ω_n that it *is* a preorder.

$$\vdash \forall_{n:\omega} \forall_{r:\omega_n \rightarrow \omega_n \rightarrow \text{t}} .(\mathsf{preorder}_n\ r) = \forall_{i,j,k:\omega_n} .(i\ r\ i \wedge ((i\ r\ j \wedge j\ r\ k) \rightarrow i\ r\ k))$$

Below, we will see that DRs which are "higher" in the resolution preorder are "better" candidates for subsequent definite anaphora.

The function $\star_n : \rho_n \rightarrow \rho_{(\textbf{suc}\ n)}$ is used to extend a resolution to the next larger domain, subject to the following schemata:

$$\vdash \forall_{n:\omega} \forall_{r:\rho_n} .n\ (\star_n\ r)\ n$$
$$\vdash \forall_{n:\omega} \forall_{r:\rho_n} \forall_{l,m:\omega_n} .(\neg(m\ (\star_n\ r)\ n)) \wedge (l\ (\star_n\ r)\ m = l\ r\ m)$$

[4] At this stage, we omit Roberts' moves, domain goals, QUD stack, etc.

Resolution extension thus occurs in a noncommittal way: for a resolution $r : \rho_n$, the extended resolution $(\star_n\ r)$ has n reflexively as high as itself, but leaves n incomparable to every $m < n$.

Information structures combine an n-ary anchor and resolution with a proposition, the common ground of the discourse. The type σ_n, mnemonic for *structure*, is a triple defined as:

$$\sigma_n =_{\text{def}} \alpha_n \times \rho_n \times \mathrm{p}$$
$$\sigma =_{\text{def}} \coprod_n \sigma_n$$

The type σ combines all the types σ_n together as a single type. The type σ plays a role analogous to that of γ (left contexts) in de Groote's type-theoretic dynamics [4], but enriches his notion of left context to include salience and a common ground in addition to a set of DRs.[5]

The function $\mathbf{next}_n : \sigma_n \to \omega$ gives the length (size of the domain) of the anchor of an n-ary structure:

$$\vdash \forall_{n:\omega}\forall_{s:\sigma_n}.(\mathbf{next}_n\ s) = n$$

In dynamic interpretations, the size of an anchor's domain is used as the "next" DR.

The functions $\mathbf{a} : \sigma \to \alpha$ (for *anchor*), $\mathbf{r} : \sigma \to \rho$ (for *resolution*) and $\mathbf{c} : \sigma \to \mathrm{p}$ (for *common ground*) are just the projections from σ to its three components. As an abbreviation, we write $[n]_s$ to denote the entity $(\mathbf{a}\,s\,n)$ that is the image of the DR n under the anchor of the structure s. When no confusion can arise, we usually drop the subscript s and write simply $[n]$.

To extend a structure with a new entity (i.e., introduce a new discourse referent and anchor it to a certain entity), we use the function $::_n : \sigma_n \to \mathrm{e} \to \sigma_{(\mathbf{suc}\ n)}$:

$$\vdash \forall_{n:\omega}.\ ::_n = \lambda_{sx}\,\langle(\mathbf{a}\,s) \bullet_n x, \star_n\,(\mathbf{r}\ s), (\mathbf{c}\ s)\rangle$$

This enriched replacement for de Groote's :: extends both a structure's anchor and its resolution, whereas de Groote's version just adds an entity to an existing set of entities. The function $+ : \sigma \to \mathrm{p} \to \sigma$ adds the ability to update the common ground of a structure with a new proposition:

$$\vdash + = \lambda_{sp}\,\langle(\mathbf{a}\,s), (\mathbf{r}\ s), (\mathbf{c}\ s) \wedge p\rangle$$

where $\wedge$ is conjunction or propositions, not truth values. Together, these two functions share the work of adding a DR to a context (::) and mutually-accepted information about a context's DRs (+). They play a central role in the definitions of the dynamic existential quantifier EXISTS and the dynamic counterparts of static propositions, discussed below. We often omit parentheses around applications involving :: and + since they both associate to the left.

[5] Actually, de Groote's theory has no DRs as distinct from entities, but we have been unable to see how to manage without such a distinction.

3.2 Continuations and Dynamic Semantics

The type κ of continuations is the type of functions from structures to propositions:

$$\kappa =_{\mathrm{def}} \sigma \to \mathrm{p}$$

Modulo replacement of de Groote's γ (left contexts) and o (truth values) by σ and p respectively, our continuations are direct analogs of his right contexts $(\gamma \to o)$. The **null continuation** is $\lambda_s \mathsf{true}$.

We use the following to recursively notate n-ary static properties, for each $n : \omega$:

$$\mathrm{r}_0 =_{\mathrm{def}} \mathrm{p}$$
$$\mathrm{r}_{(\mathbf{suc}\ n)} =_{\mathrm{def}} \mathrm{e} \to \mathrm{r}_n$$

Note that nullary static properties are simply (static) propositions. A **dynamic proposition** (type π), also known as an **update**, maps a structure and a continuation to a (static) proposition:

$$\pi =_{\mathrm{def}} \sigma \to \kappa \to \mathrm{p}$$

This is a direct analog of de Groote's type Ω. Extending Muskens [14, 15], we define the type of n-ary **dynamic relations** in an way analogous to static properties:

$$\delta_0 =_{\mathrm{def}} \pi$$
$$\delta_{(\mathbf{suc}\ n)} =_{\mathrm{def}} \omega \to \delta_n$$

In terms of their types, the difference between static and dynamic properties is that the base type for dynamic properties is π rather than p, and the type of arguments to dynamic properties is ω (of DRs) rather than e (of entities). Note that nullary dynamic properties are dynamic propositions. We abbreviate δ_1, the type of unary dynamic properties, as simply δ.

The **dynamicizer** functions **dyn** take an n-ary static property to its dynamic counterpart:

$$\vdash \mathbf{dyn}_0 = \lambda_{psk}.p \wedge (k\ (s + p)) : \mathrm{r}_0 \to \delta_0$$
$$\vdash \forall_{n:\omega}.\mathbf{dyn}_{(\mathbf{suc}\ n)} = \lambda_{Rn}.\mathbf{dyn}_n\ (R\ [n]) : \mathrm{r}_{(\mathbf{suc}\ n)} \to \delta_{(\mathbf{suc}\ n)}$$

This definitions of the **dyn** functions reflect the fact that utterances in natural language (modeled in our theory by dynamic propositions) update the discourse context with their content. As a mnemonic, we abbreviate the dynamicization of a static property by the same name as its static counterpart except that the name is written in smallcaps. For instance:

$$\vdash \textsc{rain} = (\mathbf{dyn}_0\ \mathsf{rain}) = \lambda_{sk}.\mathsf{rain} \wedge (k\ (s + \mathsf{rain}))$$
$$\vdash \textsc{snow} = (\mathbf{dyn}_0\ \mathsf{snow}) = \lambda_{sk}.\mathsf{snow} \wedge (k\ (s + \mathsf{snow}))$$
$$\vdash \textsc{donkey} = (\mathbf{dyn}_1\ \mathsf{donkey}) = \lambda_{nsk}.(\mathsf{donkey}\ [n]) \wedge (k\ (s + (\mathsf{donkey}\ [n])))$$
$$\vdash \textsc{bray} = (\mathbf{dyn}_1\ \mathsf{bray}) = \lambda_{nsk}.(\mathsf{bray}\ [n]) \wedge (k\ (s + (\mathsf{bray}\ [n])))$$
$$\vdash \textsc{own} = (\mathbf{dyn}_2\ \mathsf{own}) = \lambda_{nmsk}.(\mathsf{own}\ [n]\ [m]) \wedge (k\ (s + (\mathsf{own}\ [n]\ [m])))$$

These examples show how dynamic propositions and properties interact with the structure of the utterance context they are situated inside. The common ground is always updated by a dynamic meaning via + with the proffered content (cf. [20]) and passed to the rest of the discourse (in the form of the continuation k). In the case of the dynamic properties DONKEY, BRAY and OWN, these expect an argument that is not an entity but instead a natural number (i.e., DR) which is mapped to an entity by the anchor ($\mathbf{a}\, s$).

The static propositional content of a dynamic proposition in context is retrieved using the **staticizer** function $\mathbf{stat} : \sigma \to \pi \to \mathrm{p}$, a direct analog of de Groote's READ [5]:

$$\vdash \mathbf{stat} = \lambda_{sA}.A\; s\; \lambda_s \mathsf{true}$$

This function gives the dynamic proposition A access to the context s, but then "throws away" the rest of the discourse by specifying the null continuation as its κ-type argument. For example, assuming that the context $s : \sigma$ is such that $[n]_s = x$ for some $n : \omega$, we calculate the static content of ($\textsc{donkey}\, n$) as follows:

$$\begin{aligned}
\mathbf{stat}\; s\; (\textsc{donkey}\; n) &= \mathbf{stat}\; s\; \lambda_{sk}.(\mathsf{donkey}\; [n]) \wedge (k\; (s + (\mathsf{donkey}\; [n]))) \\
&= \mathbf{stat}\; s\; \lambda_{sk}.(\mathsf{donkey}\; x) \wedge (k\; (s + (\mathsf{donkey}\; x))) \\
&= \lambda_{sk}((\mathsf{donkey}\; x) \wedge (k\; (s + (\mathsf{donkey}\; x))))\; s\; \lambda_s \mathsf{true} \\
&= \lambda_{k}((\mathsf{donkey}\; x) \wedge (k\; (s + (\mathsf{donkey}\; x))))\; \lambda_s \mathsf{true} \\
&= (\mathsf{donkey}\; x) \wedge (\lambda_s \mathsf{true}\; (s + (\mathsf{donkey}\; x))) \\
&= (\mathsf{donkey}\; x) \wedge \mathsf{true} \\
&\equiv \mathsf{donkey}\; x
\end{aligned}$$

where $\equiv$ denotes propositional equivalence (mutual entailment). This example shows why **stat** is not defined for every structure s and dynamic proposition A, only those where A can be resolved to a static proposition based on the contents of s. Here, DONKEY must accesses the structure passed to **stat** to determine the entity that its anchor maps n to.

Dynamic Conjunction. For conjoining dynamic propositions, we likewise follow de Groote in defining dynamic AND : $\pi \to \pi \to \pi$ to compose the meanings of two dynamic propositions over a structure and a discourse continuation:

$$\vdash \textsc{and} = \lambda_{ABsk}.A\; s\; (\lambda_s.B\; s\; k) \tag{1}$$

The continuation passed to the first conjunct A is the second conjunct B with its structure (type σ) argument abstracted over. In addition to conjoining utterances to form discourses, AND plays a central role in our dynamic indefinite, given in (3), below.

To demonstrate dynamic conjunction in action, we take $\textsc{rain} = (\mathbf{dyn}_0\ \mathsf{rain})$ and $\textsc{snow} = (\mathbf{dyn}_0\,\mathsf{snow})$. The conjunction of $\textsc{rain}$ and $\textsc{snow}$ is then as follows:

$$
\begin{aligned}
&\vdash \textsc{rain and snow} : \pi \\
&= \lambda_{sk}.\textsc{rain}\ s\ (\lambda_s.\textsc{snow}\ s\ k) \\
&= \lambda_{sk}(\lambda_{sk}(\mathsf{rain} \wedge (k\ (s + \mathsf{rain}))))\ s\ (\lambda_s.\textsc{snow}\ s\ k)) \\
&= \lambda_{sk}(\lambda_{k}(\mathsf{rain} \wedge (k\ (s + \mathsf{rain})))\ (\lambda_s.\textsc{snow}\ s\ k)) \\
&= \lambda_{sk}.\mathsf{rain} \wedge (\lambda_s(\textsc{snow}\ s\ k)\ (s + \mathsf{rain})) \\
&= \lambda_{sk}.\mathsf{rain} \wedge (\lambda_{sk}(\mathsf{snow} \wedge (k\ (s + \mathsf{snow})))\ (s + \mathsf{rain})\ k) \\
&= \lambda_{sk}.\mathsf{rain} \wedge (\lambda_{k}(\mathsf{snow} \wedge (k\ (s + \mathsf{rain} + \mathsf{snow})))\ k) \\
&= \lambda_{sk}.\mathsf{rain} \wedge \mathsf{snow} \wedge (k\ (s + \mathsf{rain} + \mathsf{snow})) \\
&= \lambda_{sk}.\mathsf{rain} \wedge \mathsf{snow} \wedge (k\ \langle(\mathbf{a}\ s), (\mathbf{r}\ s), (\mathbf{c}\ s) \wedge \mathsf{rain} \wedge \mathsf{snow}\rangle)
\end{aligned}
$$

It is important to note that, at this example shows, $\textsc{and}$ ensures that the content proffered by $\textsc{rain}$ is available in the common ground of the structure $(s + \mathsf{rain})$ that is passed to $\textsc{snow}$.

The Dynamic Existential Quantifier. Our replacement for de Groote's Σ is $\textsc{exists} : \delta \to \pi$:

$$\vdash \textsc{exists} = \lambda_{Dsk}.\exists\,\lambda_x.D\ (\mathbf{next}\ s)\ (s :: x)\ k \tag{2}$$

This version of the dynamic existential quantifier introduces a DR using :: to extend both the anchor and resolution of the current structure. We examine $\textsc{exists donkey}$ for an example of the behavior of $\textsc{exists}$:

$$
\begin{aligned}
&\vdash \textsc{exists donkey} : \delta \\
&= \lambda_{sk}.\exists\,\lambda_x.\textsc{donkey}\ (\mathbf{next}\ s)\ (s :: x)\ k \\
&= \lambda_{sk}.\exists\,\lambda_x.(\mathsf{donkey}\ [(\mathbf{next}\ s)]_{s::x}) \wedge\ (k\ (s :: x + (\mathsf{donkey}\ [(\mathbf{next}\ s)]_{s::x}))) \\
&= \lambda_{sk}.\exists\,\lambda_x.(\mathsf{donkey}\ x) \wedge\ (k\ (s :: x + (\mathsf{donkey}\ x))) \\
&= \lambda_{sk}.\exists\,\lambda_x.(\mathsf{donkey}\ x) \wedge\ (k\ \langle(\mathbf{a}\ s) \bullet x, \star\,(\mathbf{r}\ s), (\mathbf{c}\ s) \wedge (\mathsf{donkey}\ x)\rangle)
\end{aligned}
$$

Note that $[(\mathbf{next}\ s)]_{s::x}$ necessarily reduces to the entity variable x because the anchor of $(s :: x)$ always maps $(\mathbf{next}\ s)$ to x, the complexity of s itself notwithstanding.

We use $\textsc{exists}$ and $\textsc{and}$ to model the English indefinite article *a* as the dynamic generalized determiner $\textsc{a} : \delta \to \delta \to \pi$:

$$\vdash \textsc{a} = \lambda_{DE}.\textsc{exists}\ \lambda_n.(D\ n)\ \textsc{and}\ (E\ n) \tag{3}$$

This definition ensures, via $\textsc{and}$, that the scope E inherits whatever extensions are made to the structure by the restriction D. We illustrate the effects of the indefinite article A by applying it to DONKEY = ($\mathbf{dyn}_1$ donkey) to yield:

$$\begin{aligned}
&\vdash \text{A DONKEY} : \delta \to \pi \\
&= \lambda_E.\text{EXISTS}\ \lambda_n.(\text{DONKEY}\ n)\ \text{AND}\ (E\ n) \\
&= \lambda_{Esk}.\exists\ \lambda_x.(\lambda_n((\text{DONKEY}\ n)\ \text{AND}\ (E\ n))\ (\mathbf{next}\ s))\ (s :: x)\ k \\
&= \lambda_{Esk}.\exists\ \lambda_x.((\text{DONKEY}\ (\mathbf{next}\ s))\ \text{AND}\ (E\ (\mathbf{next}\ s)))\ (s :: x)\ k \\
&= \lambda_{Esk}.\exists\ \lambda_x.(\mathsf{donkey}\ x) \wedge (E\ (\mathbf{next}\ s)\ (s :: x + (\mathsf{donkey}\ x))\ k) \\
&= \lambda_{Esk}.\exists\ \lambda_x.(\mathsf{donkey}\ x) \wedge (E\ (\mathbf{next}\ s)\ \langle(\mathbf{a}\ s) \bullet x, \star\ (\mathbf{r}\ s), \mathbf{c}\ s \wedge (\mathsf{donkey}\ x)\rangle\ \ k)
\end{aligned}$$

Note that the sortal restriction imposed by the noun on the new DR is part of the common ground passed to the scope. We next apply A DONKEY to the dynamic property WALK = ($\mathbf{dyn}_1$ walk):

$$\begin{aligned}
&\vdash (\text{A DONKEY WALK}) : \pi \\
&= \text{EXISTS}\ \lambda_n.(\text{DONKEY}\ n)\ \text{AND}\ (\text{WALK}\ n) \\
&= \lambda_{sk}.\exists\ \lambda_x.((\text{DONKEY}\ (\mathbf{next}\ s))\ \text{AND}\ (\text{WALK}\ (\mathbf{next}\ s)))\ (s :: x)\ k \\
&= \lambda_{sk}.\exists\ \lambda_x.(\mathsf{donkey}\ x) \wedge (\mathsf{walk}\ x) \wedge (k\ (s :: x + (\mathsf{donkey}\ x) + (\mathsf{walk}\ x))) \\
&= \lambda_{sk}.\exists\lambda_x.(\mathsf{donkey}\ x) \wedge (\mathsf{walk}\ x) \wedge k\ \ \langle(\mathbf{a}\ s) \bullet x, \star\ (\mathbf{r}\ s), (\mathbf{c}\ s) \wedge \mathsf{donkey}\ x \wedge \mathsf{walk}\ x\rangle
\end{aligned}$$

In this example, the division of labor between EXISTS and AND in the definition of A is apparent, with EXISTS extending the anchor and resolution and AND accumulating the additions to the CG made by the dynamic properties DONKEY and WALK.

4 Modeling Definiteness

With our hyperintensional dynamic semantic theory in place, we are ready to extend it to handle definiteness presuppositions in English. We examine both definite pronominal anaphora with *it* and the definite determiner *the*.

4.1 Definite Anaphora with *it*

Rather than adopting an analog of de Groote's `sel` to model English *it*, which cannot possibly select the "right" DR from a left context (see Sect. 1, above), we define dynamic IT : $\delta \to \pi$ as follows:

$$\vdash \text{IT} = \lambda_{Ds}.D\ (\mathbf{def}\ s\ \text{NONHUMAN})\ s \tag{4}$$

where NONHUMAN = ($\mathbf{dyn}_1$ nonhuman) and $\mathbf{def}_n : \sigma_n \to \delta \to \omega_n$ is the definiteness operator, defined as follows:

$$\vdash \mathbf{def}_n = \lambda_{sD}.\bigsqcup_{(\mathbf{r}\ s)} \lambda_{i:\omega_n}.(\mathbf{c}\ s)\ \mathsf{entails}\ (\mathbf{stat}\ s\ (D\ i)) \tag{5}$$

(Recall that $\mathsf{entails} : \mathrm{p} \to \mathrm{p} \to \mathrm{t}$ is the entailment relation between (static) propositions.) For each $r : \rho_n$, the operator $\bigsqcup_r : (\omega_n \to \mathrm{t}) \to \omega_n$ takes a subset of the first n DRs and returns the unique greatest element (if any) with respect to the (restriction of the) preorder r on ω_n. Thus for a structure s and a dynamic property D, the **def** operator returns the highest DR (if any) in the resolution $(\mathbf{r}\, s)$ whose image under the current anchor $(\mathbf{a}\, s)$ can be inferred from the common ground $(\mathbf{c}\, s)$ to have the staticized counterpart of the property D.

As defined in (4), IT selects the most salient inferably NONHUMAN discourse referent from a given structure. This is because, as (5) shows, IT is equivalent to

$$\lambda_{Ds}.D\,(\bigsqcup_{(\mathbf{r}\,s)} \lambda_{i:\omega_n}((\mathbf{c}\,s)\,\mathsf{entails}\,(\mathsf{nonhuman}\,(\mathbf{a}\,s\,i))))\;s$$

We can assume that the static proposition (every donkey nonhuman) reflecting the common world knowledge that every donkey is nonhuman is in every common ground we would consider, where every is as given in [19]. This ensures that **def** will allow IT to select a donkey as its antecedent, as desired.

We demonstrate by applying this definition of IT in the interpretation of the following example:

(I) 1. A donkey enters.
    2. It brays.

With DONKEY $= (\mathbf{dyn}_1 \mathsf{donkey})$, ENTER $= (\mathbf{dyn}_1 \mathsf{enter})$, and BRAY $= (\mathbf{dyn}_1 \mathsf{bray})$, we analyze (I) as:

$$\vdash (\text{A DONKEY ENTER})\ \text{AND}\ (\text{IT BRAY}) : \pi$$

To see how IT retrieves its antecedent from context, we examine the rightmost application:

$$\begin{aligned} &\vdash \text{IT BRAY} : \pi \\ &= \lambda_s.\text{BRAY}\,(\mathbf{def}\; s\ \text{NONHUMAN})\; s \\ &= \lambda_s.\lambda_{nsk}((\mathsf{bray}\,[n]) \wedge (k\,(s + (\mathsf{bray}\,[n]))))\,(\mathbf{def}\; s\ \text{NONHUMAN})\; s \\ &= \lambda_{sk}.(\mathsf{bray}\,[(\mathbf{def}\; s\ \text{NONHUMAN})]) \wedge (k\,(s + (\mathsf{bray}\,[(\mathbf{def}\; s\ \text{NONHUMAN})]))) \end{aligned} \tag{6}$$

Here, IT ensures that the argument to BRAY is the most salient nonhuman entity in the discourse context. Recall from the example analysis given above of (A DONKEY WALK) that A DONKEY updates the common ground passed to IT BRAY with the proposition that $[n]$ is a donkey. The full reduction of the dynamic interpretation of (I) in is then:

$$\begin{aligned} &\vdash (\text{A DONKEY ENTER})\ \text{AND}\ (\text{IT BRAY}) : \pi \\ &= (\text{EXISTS}\ \lambda_n((\text{DONKEY}\ n)\ \text{AND}\ (\text{ENTER}\ n)))\ \text{AND}\ (\text{IT BRAY}) \\ &= \lambda_{sk}.\exists\,\lambda_x.(\mathsf{donkey}\ x) \wedge (\mathsf{enter}\ x) \\ &\qquad \wedge (\mathsf{bray}\,[(\mathbf{def}\ \varsigma\ \text{NONHUMAN})]) \wedge (k\,(\varsigma + (\mathsf{bray}\,[(\mathbf{def}\ \varsigma\ \text{NONHUMAN})]))) \end{aligned} \tag{7}$$

where $\varsigma = s :: x + (\mathsf{donkey}\, x) + (\mathsf{enter}\, x)$ is the structure passed to IT BRAY (shown in (6)). Since the CG of ς contains the proposition $(\mathsf{donkey}\ x)$, the definiteness operator **def** is able to select the DR that is the preimage of x as the most salient nonhuman in the context it is passed. With $[(\mathbf{def}\ \varsigma\ \text{NONHUMAN})]_\varsigma = x$, we can reduce the term in (7) interpreting the discourse in (I) to:

$$\lambda_{sk}.\exists\, \lambda_x.(\mathsf{donkey}\ x) \land (\mathsf{enter}\ x) \land (\mathsf{bray}\ x) \land (k\ (\varsigma + (\mathsf{bray}\ x))) : \pi$$

Thus IT selects its antecedent based on its definiteness presuppositions, yielding the desired truth conditions for (I). The definition of IT in (4) also captures the infelicity of (C), where *it* is used without a salient antecedent, because there is no DR that can be inferred from context to be nonhuman.

4.2 The Definite Determiner

We also use **def** to model the English definite determiner *the* as $\text{THE} : \delta \to \delta \to \pi$:

$$\vdash \text{THE} = \lambda_{DEs}.(\lambda_n((D\,n)\ \text{AND}\ (E\,n))\ (\mathbf{def}\ s\ D))\ s \tag{8}$$

This translation of *the* resembles the indefinite determiner A in (3) in that the meanings of the dynamic properties D and E are composed via AND. The main difference is that while A uses EXISTS to introduce a new DR, THE uses **def** from (5) to select the most salient DR from the discourse context with the property D. Using AND to pass this DR to both properties ensures that any modifications to the structure that result from D are inherited by E, as would be necessary in the interpretation of an utterance like *The donkey that has a red blanket chews it.*

In the discourse in (J), which is a simplification of (A), (B), and (F), the noun phrase *the donkey* can only refer to one of the discourse referents introduced prior to its use:

(J)
1. A donkey enters.
2. A mule enters.
3. The donkey brays.

To model this discourse, we first define the dynamic properties $\text{MULE} = (\mathbf{dyn}_1\ \mathsf{mule})$ and $\text{ENTER} = (\mathbf{dyn}_1\ \mathsf{enter})$. With A and THE as in (3) and (8), and AND as in (1) used to conjoin utterances, we have the following dynamic meaning for the discourse in (J):

$$\vdash ((\text{A DONKEY ENTER})\ \text{AND}\ (\text{A MULE ENTER}))\ \text{AND}\ (\text{THE DONKEY BRAY}) : \pi \tag{9}$$

We start with the leftmost conjunct of the discourse:

$$\begin{aligned} &\vdash (\text{A DONKEY ENTER}) : \pi \\ &= \lambda_{sk}.\exists\, \lambda_x.(\mathsf{donkey}\ x) \land (\mathsf{enter}\ x) \land (k\ (s :: x + (\mathsf{donkey}\ x) + (\mathsf{enter}\ x))) \\ &= \lambda_{sk}.\exists\, \lambda_x.(\mathsf{donkey}\ x) \land (\mathsf{enter}\ x) \\ &\qquad \land (k\ \langle (\mathbf{a}\, s) \bullet x, \star\, (\mathbf{r}\ s), (\mathbf{c}\ s) \land (\mathsf{donkey}\ x) \land (\mathsf{enter}\ x)\rangle) \end{aligned} \tag{10}$$

Combining the term in (10) with the entire left conjunct of the discourse, we have:

$$\begin{aligned} &\vdash (\text{A DONKEY ENTER}) \ \text{AND} \ (\text{A MULE ENTER}) : \pi && (11)\\ &= \lambda_{sk}.\exists\, \lambda_x.(\mathsf{donkey}\ x) \wedge (\mathsf{enter}\ x) \wedge (\exists\, \lambda_y.(\mathsf{mule}\ y) \wedge (\mathsf{enter}\ y) \wedge (k\,\varsigma)) \end{aligned}$$

where ς represents the structure that results from the application of AND in (11):

$$\begin{aligned} \varsigma &= s :: x + (\mathsf{donkey}\ x) + (\mathsf{enter}\ x) :: y + (\mathsf{mule}\ y) + (\mathsf{enter}\ y)\\ &= \langle(\mathbf{a}\ s) \bullet x \bullet y, \star\,(\star\,(\mathbf{r}\ s)), (\mathbf{c}\ s) \wedge (\mathsf{donkey}\ x) \wedge (\mathsf{enter}\ x) \wedge (\mathsf{mule}\ y) \wedge (\mathsf{enter}\ y)\rangle \end{aligned}$$

The right conjunct then uses THE to select the most salient DONKEY from the preceding discourse, applying BRAY to it:

$$\begin{aligned} &\vdash (\text{THE DONKEY BRAY}) : \pi && (12)\\ &= \lambda_{sk}.(\lambda_n((\text{DONKEY}\ n) \ \ \text{AND} \ \ (\text{BRAY}\ n))\ (\mathbf{def}\ s\ \text{DONKEY}))\ s\ k\\ &= \lambda_{sk}.((\text{DONKEY}\ (\mathbf{def}\ s\ \text{DONKEY}))\ \text{AND}\ (\text{BRAY}\ (\mathbf{def}\ s\ \text{DONKEY})))\ s\ k\\ &= \lambda_{sk}.(\mathsf{donkey}\ [(\mathbf{def}\ s\ \text{DONKEY})]) \wedge (\mathsf{bray}\ [(\mathbf{def}\ s\ \text{DONKEY})])\\ &\qquad \wedge (k\ (s + (\mathsf{donkey}\ [(\mathbf{def}\ s\ \text{DONKEY})]) + (\mathsf{bray}\ [(\mathbf{def}\ s\ \text{DONKEY})]))) \end{aligned}$$

The structure ς passed to (THE DONKEY BRAY) by the preceding discourse in (11) is such that $[(\mathbf{def}\,\varsigma\,\text{DONKEY})]_\varsigma = x$ because the CG of ς contains the proposition $(\mathsf{donkey}\,x)$ but does not contain any proposition in which the property donkey is applied to an entity other than x. Hence we arrive at the final reduction of the term in (9) that models the entirety of the discourse (J):

$$\lambda_{sk}.\exists\,\lambda_x.\mathsf{donkey}\ x \wedge \mathsf{enter}\ x \wedge (\exists\,\lambda_y.\mathsf{mule}\ y \wedge \mathsf{enter}\ y \wedge \mathsf{donkey}\ x \wedge \mathsf{bray}\ x \wedge k\ \varsigma')$$

where $\varsigma' = \varsigma + (\mathsf{donkey}\,x) + (\mathsf{bray}\,x)$ is the structure extending ς that results from applying AND to the left (11) and right (12) conjuncts of the discourse.

Note that, in this example, the dynamic definite determiner THE picks out the most salient DR from the structure it is given that has the property specified as its argument (i.e., DONKEY). However, as (F3) shows, substituting the pronoun *it* for *the donkey* makes the discourse infelicitous. Our theory captures this infelicity because IT only requires its antecedent to have the property $\mathsf{nonhuman}$, which is weaker than the property donkey with respect to entailment. With IT replacing THE DONKEY in the right conjunct of (9), **def** would be incapable of selecting a unique nonhuman DR from ς since two DRs would then have the property NONHUMAN (namely, the mule and the donkey). The infelicitous examples in (E) through (H) can be ruled out for similar reasons.

5 Conclusion and Future Work

We have presented a dynamic theory of discourse meaning formulated in higher order logic that incorporates aspects of de Groote's continuation-based theory and Pollard's hyperintensional semantics. Drawing on the work of Heim and Roberts, our theory provides an enriched notion of discourse context that includes discourse referents ordered by relative salience and a common ground of mutually accepted content. We have shown how this enriched context allows the definiteness presuppositions in English associated with the pronoun *it* and the determiner *the* to be captured in a way that is faithful to the facts. The resulting theory repairs the inadequate treatment of anaphora resolution in de Groote's work based on the oracular `sel` function.

In future work, we will continue our collaboration with Roberts and Smith on developing a general, categorially based theory of projective meaning. The next avenues for this research include spelling out how relative salience is adjusted by re-invoking a previously introduced DR (see the discussion of (A) and (B) in Sect. 1) and integrating the hyperintensional dynamic semantic theory introduced here with a fully compositional theory of English grammar that takes e.g. quantifier scope, unbounded dependencies, and prosodically encoded information structure into account. We will then apply this theory to a wider range of presuppositional phenomena, including the factivity of certain sentential complement verbs (e.g. *suck*, *know*, *realize*), the 'projection' of presuppositions occurring within e.g. the scope of a negation, and the phenomenon known as *farmer-donkey asymmetry* [23] that is associated with the celebrated 'donkey sentences' (e.g. *Most farmers who own a donkey beat it*).

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